Chapter 7 HW 2012

Saturday, August 25, 2012
10:40 PM

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a

$$P = \frac{100000 A_{60}}{\ddot{a}_{60}} = \frac{100,000(.36913)}{11.1454} = 3311.959$$

$${}_{10}V = PVFB - PVFP$$

$$= 100,000 A_{70} - 3311.959\, \ddot{a}_{70}$$

$$= 51495 - 28381.09 =$$

$$= \boxed{23,113.91}$$

b

$$P = \frac{250,000 A^{1}_{40:\overline{20|}}}{\ddot{a}_{40:\overline{20|}}} =$$

$$\frac{250,000(A_{40} - {}_{20}E_{40} A_{60})}{\ddot{a}_{40} - {}_{20}E_{40}\, \ddot{a}_{60}} =$$

$$\frac{250,000(0.16132 - (.27414)(.36913))}{14.8166 - (.27414)(11.1454)}$$

$$= 1278.0733$$

$${}_{10}V = PVFB - PVFP$$

$$= 250000 A^{1}_{50:\overline{10|}} - 1278.0733\, \ddot{a}_{50:\overline{10|}}$$

$$= 250,000(.24905 - (.51081)(.36913))$$

$$- 1278.0733\left(13.2668 - (.51081)(11.1454)\right)$$

$$= \boxed{5444.04}$$

(c) $PVP = PVB$

$$P\ddot{a}_{35:\overline{10}|} = 10{,}000\, A_{35:\overline{30}|}$$

$$P = \frac{10000\left(A_{35} - {}_{30}E_{35}\, A_{65} + {}_{30}E_{35}\right)}{\ddot{a}_{35} - {}_{10}E_{35}\, \ddot{a}_{45}}$$

$$= \frac{10000\left(.12872 - (.286)(.48686)(.43980 - 1)\right)}{15.3926 - (.54318)(14.1121)}$$

$$= 267.52$$

$${}_5V = 10000\, A_{40:\overline{25}|} - 267.52\, \ddot{a}_{40:\overline{5}|}$$

$$= 10{,}000\left(.16132 - (.27414)(.68756)(.43980) + (.27414)(.68756)\right)$$

$$- 267.52\left(14.8166 - (.73529)(14.1121)\right)$$

$$= 2669.11 - 1187.82 = \boxed{1481.29}$$

$${}_{10}V = 10{,}000\, A_{45:\overline{20}|} - 0$$ ← There are no more premiums

$$= 10{,}000\left(.20120 - (.25634)(.43980) + .25634\right)$$

$$= \boxed{3448.02}$$

(d) ${}_{10}V = 1000\,\ddot{a}_{75} = \boxed{7217}$ before the payment. After the payment $= 7217 - 1000 = \boxed{6217}$

(e) PVP = PVB

$$P\ddot{a}_{60} = 50{,}000\,\bar{A}_{60}$$

$$P = \frac{50{,}000(1.02971)(.36913)}{11.1454}$$

$$= 1705.1737$$

$${}_{10}V = PVFB - PVFP$$
$$= 50000\,\bar{A}_{70} - 1705.1737\,\ddot{a}_{70}$$

$$= 50000(1.02971)(.51495) - 1705.1737(8.5693)$$

$$= \boxed{11{,}900.31}$$

(f) First find net premium. Let P be monthly premium. Then

$$12P\ddot{a}^{(12)}_{60} = 50{,}000\,\bar{A}_{60}$$

$$12P = \frac{50000(1.02971)(.36913)}{(1.00028)(11.1454) - .46812}$$

$$= 1779.4129$$

$${}_{10}V = 50000\,\bar{A}_{70} - 12P\ddot{a}^{(12)}_{70}$$

$$= 50000(1.02971)(.51495) - (1779.4129)((1.00028)(8.5693) - 0.46812)$$

[illegible]

$$= \boxed{12{,}092.841}$$

(g) $${}_{10}V^{FPT} = PVB - PV\,P_{x+1}$$

$$= 100{,}000\,A_{70} - \frac{A_{61}}{\ddot{a}_{61}}\,\ddot{a}_{70}$$

$$= 100{,}000(0.51495) - \frac{.38279(100{,}000)}{10.9041}(8.5693)$$

$$= 21{,}412.35$$

(h) $${}_{10}V^{FPT} = PVB - PV\,P_{x+1}$$

$$= 250{,}000\left(A_{50} - {}_{10}E_{50}\,A_{60}\right)$$

$$- 250{,}000\left(\frac{A_{41} - {}_{19}E_{41}\,A_{60}}{\ddot{a}_{41} - {}_{19}E_{41}\,\ddot{a}_{60}}\right)\left(\ddot{a}_{50} - {}_{10}E_{50}\,\ddot{a}_{60}\right)$$

$$= 250{,}000\left(0.24905 - (0.51081)(0.36913)\right)$$

$$- 250{,}000\left(\frac{0.16869 - (1.06)^{-19}\left(\frac{8188074}{9{,}287{,}264}\right)(.36913)}{14.6864 - (1.06)^{-19}\left(\frac{8{,}188{,}074}{9{,}287{,}264}\right)(11.1454)}\right)\cdot$$

$$\left(13.2668 - (0.51081)(11.1454)\right)$$

$$= 15{,}123.68 - (1335.97672)(7.57362)$$

$$= 5005.50$$

(i) $${}_{5}V^{FPT} = PVB - PV\,P_{x+1}$$

$$P_{x+1} = \frac{A_{36} - {}_{29}E_{36}\,A_{65} + {}_{29}E_{36}}{\ddot{a}_{36} - {}_{9}E_{36}\,\ddot{a}_{45}}(10{,}000)$$

$$= 10{,}000\left[(.13470) - (1.06)^{-29}\left(\frac{7{,}533{,}964}{\ldots}\right)(.43980 - 1)\right]$$

$$10{,}000\left[\frac{\quad\cdots\quad 9{,}401{,}688)\quad}{15.2870-(1.06)^{-9}\left(\frac{9{,}164{,}051}{9{,}401{,}688}\right)(14.1121)}\right]$$

$$=304.46973$$

$${}_5V^{FPT}=10{,}000\left[A_{40}-{}_{25}E_{40}(A_{65}-1)\right]$$

$$-304.46973\left(\ddot{a}_{40}-{}_5E_{40}\,\ddot{a}_{45}\right)$$

$$=10{,}000\left[.16132-(.27414)(.68756)(.43980-1)\right]$$

$$-304.46973\left[14.8166-(.73529)(14.1121)\right]$$

$$=\boxed{1337.23}$$

$${}_{10}V^{FPT}=PVB \quad \text{since no more premiums}$$

$$=10{,}000\left(A_{45}-{}_{20}E_{45}(A_{65}-1)\right)$$

$$=10{,}000\left[0.20120-(0.25634)(.43980-1)\right]$$

$$=\boxed{3448.02}$$

85 (a) $PVP = PVB + PVE$

$$P\ddot{a}_{60}=100{,}000A_{60}+.42P+.08P\ddot{a}_{60}$$

$$+100+25\ddot{a}_{60}+250A_{60}$$

$$P=\frac{100250(.36913)+100+25(11.1454)}{.92(11.1454)-.42}$$

$$=3801.5863$$

$${}_{10}V=PVFB+PVFE-PVFP$$

$$=100{,}250(A_{70})+(.08)(3801.5863)\ddot{a}_{70}$$

$$25\ddot{a}_{70} - 3801.5863\,\ddot{a}_{70}$$

$$= 100{,}250(.51495) -$$

$$\left[(.92)(3801.5863) - 25\right](8.5693)$$

$$= \boxed{21{,}867.19}$$

(b) ${}_0V = PVFB + PVFE - PVFP$

$$= 250{,}000\, A^{\,1}_{40:\overline{20|}} + (.42)(1600) +$$

$$(.08)(1600)\,\ddot{a}_{40:\overline{20|}} + 100 + 25\,\ddot{a}_{40:\overline{20|}}$$

$$+ 250\, A^{\,1}_{40:\overline{20|}} - 1600\,\ddot{a}_{40:\overline{20|}}$$

$$= 250250\left(.16132 - (.27414)(.36913)\right)$$

$$+ 672 + 100 -$$

$$\left[.92(1600) - 25\right]\left(14.8166 - (.27414)(11.1454)\right)$$

$$= \boxed{-1199.75}$$

$${}_{10}V = 250{,}000\, A^{\,1}_{50:\overline{10|}} + (.08)(1600)\,\ddot{a}_{50:\overline{10|}}$$

$$+ 25\,\ddot{a}_{50:\overline{10|}} + 250\, A^{\,1}_{50:\overline{10|}} - 1600\,\ddot{a}_{50:\overline{10|}}$$

$$= 250{,}250\left(.24905 - (.51081)(.36913)\right)$$

$$- \left[(.92)(1600) - 25\right]\left(13.2668 - (.51081)(11.1454)\right)$$

$$= \boxed{4179.77}$$

(c) $P\ddot{a}_{35:\overline{10|}} = 10000 A_{35:\overline{20|}} + .42P +$

$.08P\ddot{a}_{35:\overline{10|}} + 100 + 25\,\ddot{a}_{35:\overline{20|}}$

$+ 250 A_{35:\overline{20|}}$

$$P = \frac{10250(.12872 - (.286)(.30514) + .286) + 100 + 25(15.3926 - (.286)(12.2758))}{(.92)(15.3926 - (.54318)(4.1121)) - .42}$$

$= 561.13$

$G = 561.13 + 50 = 611.13$

${}_5V = 10280\, A_{40:\overline{15|}} + 25\,\ddot{a}_{40:\overline{15|}}$

$+ .08(611.13)(\ddot{a}_{40:\overline{5|}}) - 611.13\,\ddot{a}_{40:\overline{5|}}$

$= 10250(.16132 - (.53667)(.72137)(.30514)$

$+ (.53667)(.72137))$

$+ 25(14.8166 - (.53667)(.72137)(12.2758))$

$-(.92)(611.13)\left[14.8166 - (.73529)(4.1121)\right]$

$= \boxed{2166.04}$

${}_{10}V = 10250\, A_{45:\overline{10|}} + 25\,\ddot{a}_{45:\overline{10|}}$

$= 10250\left[.20120 - (.52652)(.30514)\right.$

$+ .52652] + 25[14.1121 -$

$(.52652)(12.2758)]]$

$= \boxed{6003.56}$

(86) $_tV^n = 1000\bar{A}_{x+t} - P\bar{a}_{x+t}$

$P = \frac{1000\bar{A}_x}{\bar{a}_x}$; $\bar{a}_{x+t} = \frac{1-\bar{A}_{x+t}}{d}$

$\bar{a}_x = \frac{1-\bar{A}_x}{d}$

$\Rightarrow {}_tV^n = 1000\bar{A}_{x+t} - \frac{1000\bar{A}_x}{\bar{a}_x}\bar{a}_{x+t}$

$= 500 - 400\left(\frac{\frac{1-\bar{A}_{x+t}}{d}}{\frac{1-\bar{A}_x}{d}}\right)$

$= 500 - 400\,\frac{1-.5}{1-.4}$

$= 500 - 400(5/6) = \boxed{166.67}$

(87) $_tV^n = 1000\bar{A}_{x+t} - P\bar{a}_{x+t}$

$P = \frac{1000\bar{A}_x}{\bar{a}_x}$; $\bar{A}_x = 1-\delta\bar{a}_x$

$$\bar{A}_{x+t} = 1 - \delta\, \bar{a}_{x+t}$$

$$\Rightarrow 1000\left(1-\delta\,\bar{a}_{x+t}\right) - \frac{1000(1-\delta\bar{a}_x)}{\bar{a}_x}\,\bar{a}_{x+t}$$

$$= 1000 - \delta(1000)(8.4) - \frac{1000(1-\delta(12))}{12}(8.4)$$

$$= 1000 - \delta(1000)(8.4) - \frac{1000(8.4)}{12} + 1000(\delta)(8.4)$$

$$= 1000 - \frac{1000(8.4)}{12} = \boxed{300}$$

88 $\quad {}_0V = 0$

$${}_{t+1}V = \frac{({}_tV+P_t)(1+i) - q_{x+t}(S_{t+1})}{p_{x+t}}$$

$${}_1V = \frac{(0+3736.756)(1.04)-(.2)(10000)}{.8}$$

$$= \boxed{2357.78}$$

$${}_2V = \frac{(2357.78+3736.756)(1.04) - (.4)(10000)}{.6}$$

$$= \boxed{3897.20}$$

$${}_3V = \frac{(3897.20+3736.756)(1.04) - (.5)(10000)}{}$$

$$\overline{\qquad\qquad\qquad\qquad}$$
$$.5$$

$$= \boxed{5878.63}$$

$_4V = 0$

(89) $PVP = PVB$

$$2P(900) + 2P(720)v + P(432)v^2 + P(216)v^3$$

$$= 1660(180)v + 1000(288)v^2 + 500(216)v^3 + 500(216)v^4$$

$$P = \frac{627{,}679.5718}{3776.04688} = 166.226636$$

$$_2V = PVFB - PVFP$$

$$= 500v\left(\frac{216}{432}\right) + 500v^2\left(\frac{216}{432}\right) - 166.226636\left(1 + \frac{216}{432}v\right)$$

$$= \boxed{225.38}$$

(90) $P_2 = 282.235$

$P_1 = 2(P_2) = 564.470$

$P_0 = 2(P_1) = 1128.940$

$_0V = 0$

$$_1V = \frac{(_0V + P_0)(1+i) - S_1 q_x}{1 - q_x}$$

$$= \frac{(0 + 1128.940)(1.04) - 2000(.1)}{.9}$$

$$= 1082.33$$

$${}_2V = \frac{(1082.33 + 564.47)(1.04) - 2000(.2)}{.8}$$

$$= 1640.84$$

Endows

$${}_3V = (1640.84 + 282.235)(1.04) - 2000(1)$$

$$= 0$$

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$${}_{10}V = \frac{({}_9V + P)(1+i) - 1000\,q_{59}}{1 - q_{59}}$$

$$560 = \frac{(500 + 60)(1.1) - 1000\,q_{59}}{1 - q_{59}}$$

$$560 - 560\,q_{59} = 616 - 1000\,q_{59}$$

$$q_{59} = \frac{56}{440} = \boxed{\frac{7}{55}}$$

92 ${}_0V = 0$ since premium is calculated using the equivalence principle.

$${}_1V = \frac{({}_0V + P_0 - e_0 - X_0^{BOY})(1+i) - (S_0 + E_0)q_x}{1 - q_x}$$

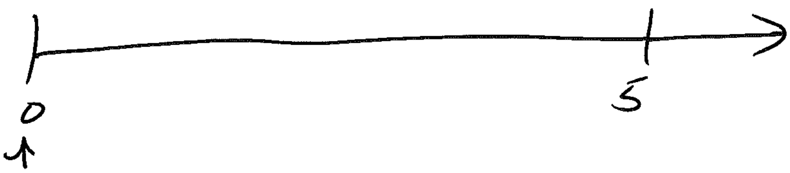
$$-79.56 = \frac{(0 + P - .5P - 100)(1.08) - (10000)(.002)}{1 - .002}$$

$$(-79.56)(.998) = .54P - 108 - 20$$

$$P = \frac{128 - 79.56(.998)}{.54} = \boxed{90}$$

(93) Since we do not know future death benefits, we cannot calculate the reserve prospectively. We must use retrospective approach.

First find present value of benefits & premiums for the first 5 years.



PVP - PVB

$= 700\,\ddot{a}_{40:\overline{5}|} - 100{,}000\,A^{1}_{40:\overline{5}|}$

$= 700\left(14.8166 - (.73529)(14.1121)\right)$

$\quad - 100{,}000\left(.16132 - (.73529)(.20120)\right)$

$= 1770.11459$

Now we have to accumulate the present value to time 5 by dividing by ${}_5E_{40}$

${}_5V = (1770.11459)\left(\frac{1}{{}_5E_{40}}\right) = \boxed{2407.37}$

(94) $P = \frac{1000A_{65}}{\ddot{a}_{65}} = \frac{439.80}{9.8969} = 44.4382$

${}_{10}V = 1000A_{75} - 44.4382\,\ddot{a}_{75}$

$= 270.78$

${}_{11}V = 1000A_{76} - 44.4382\,\ddot{a}_{76}$

$= 297.84$

$${}_{10.7}V = (1-.3)({}_{10}V + P) + (.3)({}_{11}V)$$

$$= (.3)(270.78 + 44.4382) + (.7)(297.84)$$

$$= \boxed{303.05}$$

(95) (a) $PVB + PVE = PVP$

$$P\ddot{a}_{70} = 25000 A_{70} + 0.56P + 0.04P\ddot{a}_{70} + 100 + 20\ddot{a}_{70}$$

$$P = \frac{25000 A_{70} + 100 + 20\ddot{a}_{70}}{.96\ddot{a}_{70} - 0.56}$$

$$= \frac{25,000(.51495) + 100 + 20(8.5693)}{(0.96)(8.5693) - 0.56}$$

$$= 1714.61397$$

(b) $PVB + PVE - PVP$

$$25000 A_{75} + .04(1714.61397)\ddot{a}_{75} + 20\ddot{a}_{75} - (1714.61397)\ddot{a}_{75}$$

$$= 25000(.59149) - [(.96)(1714.61397) - 20](7.2170)$$

$$= 3052.20$$

(c) $${}_{6}V = \frac{({}_{5}V + P - E)(1.06) - 25000\, q_{x+5}}{}$$

$$1 - q_{x+5}$$

$$= \frac{\left(3052.20 + (0.96)(1714.61397) - 20\right)(1.06) - 25000(0.05169)}{1 - 0.05169}$$

$$= 3866.53$$

d) $${}_6V' = \frac{\left(3052.20 + 0.96(1714.61397) - 30\right)(1.058) - 25000(.95)(.05169)}{1 - (.95)(.05169)}$$

$$= 3903.00$$

$$\text{GAIN} = 3903.00 - 3866.53$$

$$= 36.47$$

e) $${}_6V' = \frac{\left(3062.20 + .96(1714.61397) - 20\right)(1.06) - 25000(.95)(.05169)}{1 - (.95)(.05169)}$$

$$= 3923.97$$

GAIN FROM MORT

$$= 3923.97 - 3866.53 = 57.44$$

$${}_6V'' = \frac{\left(3052.20 + (.96)(1714.61397) - 20\right)(1.058) - 25000(.95)(.05169)}{1 - (.95)(.05169)}$$

$= 3914.13$

GAIN FROM INTEREST

$= 3914.13 - 3923.97 = -9.84$

GAIN FROM EXPENSE

$= 3903.00 - 3914.13 = -11.13$

Note: $57.44 - 9.84 - 11.13 = 36.47$

(96) G^m = Gain from Mortality

G^E = Gain from Expenses

$$P = \frac{50000 A_{40}}{\ddot{a}_{40}} = \frac{50(161.32)}{14.8166} = 544.39$$

so Gross = 680.49

$${}_{10}V = (50{,}000 + 300)A_{50} - (1-.05)(680.49)(\ddot{a}_{50})$$

$$= (50{,}300)(0.24908) - (.95)(680.49)(13.2668)$$

$= 3950.69$ per person

$${}_{11}V = \frac{({}_{10}V + P_t - e_t)(1+i) - q_{50}(S_{11} + E_{11})}{p_{50}}$$

$$\frac{[3950.69 + (680.49)(.95)](1.06) - 0.00592(50{,}300)}{1 - 0.00592}$$

$= 4602.46$

$${}_{11}V^m = [3950.69 + (680.49)(.95)](1.06)$$

$${}_{11}V = \frac{L \qquad - 0.005(50,300)}{1-.005}$$

$$= 4644.70$$

$$G^m = 4644.70 - 4602.46 = 42.24$$

$${}_{11}V^{m\&E} = \frac{[3950.69 + (680.49)(.94)(1.06) + (.005)(50,100)}{1-0.005}$$

$$= 4638.46$$

$$G^E = 4638.46 - 4644.70 = -6.24$$

But this is per policy and we had 1000 policies so

$$-6.24 \times 1000 = -\underline{\underline{6240}}$$

(97) ${}_0AS = \boxed{0}$

$${}_1AS = \frac{({}_0AS + P_0 - e_0 - X_0^{BOY})(1+i) - (S_1 + E_1)q_x}{1-q_x}$$

$$= \frac{(0+300-(.2)(300) - 30)(1.08) - 10000(.01)}{.99}$$

$$= \boxed{18.99}$$

$$(18.99 + 300 - (.08)(300) - 30)(1.08)$$

$$_2AS = \frac{\quad - 10000(.015)}{.985}$$

$$= \boxed{138.26}$$

$$_3AS = \frac{(138.26 + 300 - (0.08)(300) - 30)(1.08) - 10000(.02)}{.98}$$

$$= \boxed{219.39}$$

FPS

(98) $_{10}V = PVFB - PVFP$

$$= 1000\bar{A}_{50} + 1000\bar{A}^{1}_{50:\overline{10}|}$$

$$- 66\bar{a}_{50:\overline{10}|}$$

$$= 333.33 + 197.81 - 66\left(\frac{1-\bar{A}_{50:\overline{10}|}}{\delta}\right)$$

$$= 531.14 - 66\left(\frac{1 - 0.19781 - 0.46657}{.06}\right)$$

$$= 95.96$$

(99) Use the recursive formula

$$_0V = 0$$

$$_1V = \frac{(_0V + P)(1+i) - (S)(q_x)}{p_x}$$

$$= \frac{(0 + 323.63)(1.06) - 1000(.2)}{1 - .2}$$

$= 245.06$

$${}_2V = \frac{({}_1V + P)(1+i) - 1000\, q_{x+1}}{1 - q_{x+1}}$$

$$= \frac{(245.06 + 373.63)(1.06) - (1000)(.2)}{.8}$$

$= 569.76$

${}_2V - {}_1V = 569.76 - 245.06$

$= \boxed{324.70}$

(100) Use recursive formula

${}_0V = 0$

$${}_1V = \frac{({}_0V + P)(1+i) - 5\, q_x}{p_x}$$

$$= \frac{(0 + 218.15)(1.06) - (10000)(.02)}{.98}$$

$= 31.8765$

$${}_2V = \frac{(31.8765 + 218.15)(1.06) - (9000)(.021)}{1 - .021}$$

$= \boxed{77.66}$

(101) Net Premium P^n

$$= \frac{1{,}000{,}000\, A_{65}}{\ddot{a}_{65}} =$$

$$\frac{1{,}000{,}000(.43980)}{9.8969} = \boxed{44{,}438.16}$$

(b) Gross Premium $= P^g$

$PVP = PVB + PVE$

$$P^g \ddot{a}_{65} = 1{,}000{,}000 A_{65} + 60 + 40\ddot{a}_{65}$$

$$+ .47 P^g + .03 P^g \ddot{a}_{65} +$$

$$(1000)(0.9) + (1000)(0.1)\ddot{a}_{65} + 200 A_{65}$$

$$P^g = \frac{1{,}000{,}200 A_{65} + 960 + 140\ddot{a}_{65}}{\ddot{a}_{65}(.97) - .47}$$

$$= \frac{1{,}000{,}200(.43980) + 960 + 140(9.8968)}{(9.8969)(.97) - .47}$$

$$= \boxed{48{,}437.44}$$

(c) $P^e = P^g - P^n = 48{,}437.44 - 44{,}438.16$

$$= \boxed{3999.28}$$

(d) ${}_{10}V^n = 1{,}000{,}000\, A_{75} - 44{,}438.16\, \ddot{a}_{75}$

$$= 1{,}000{,}000(.59149) - 44{,}438.16(7.2170)$$

$$= \boxed{270{,}779.80}$$

(e)

(e) ${}_{10}V^e = PVFE - PVFP^e$

$= 40\ddot{a}_{25} + (.03)(48{,}437.44)\ddot{a}_{25}$

$+ 1000(.1)\ddot{a}_{25} + 200A_{25}$

$- 3999.28\,\ddot{a}_{25}$

$= 200A_{25} - (2406.16)\ddot{a}_{25}$

$= 200(.59149) - (2406.16)(7.2170)$

$= \boxed{-17{,}246.96}$

(f) ${}_tV^g = {}_tV^h + {}_tV^e =$

$270{,}779.80 - 17{,}246.96$

$= \boxed{253{,}532.84}$

(g)

$${}_1AS = \frac{({}_0AS + P^g - e_0 - X_0^{50\%})(1+i) - q_x(S_0 + E_0)}{p_x}$$

$$= \frac{[0 + 48{,}437.44 - (.5)(48437.44) - 100 - 1000(1)](1.06) - (0.02132)(1{,}000{,}200)}{1 - 0.02132}$$

$= \boxed{3250.89}$

(h)

$${}_2AS = \frac{[3250.89 + 48{,}437.44 - (.03)48437.44 - 40 - (1000)(0.10)](1.06) - (1{,}000{,}200)(0.02329)}{1 - 0.02329}$$

$$= \boxed{30{,}517.00}$$

(102)

$$PVP = PVB + PVE$$

$$P(1 + (.99)(v) + (.99)(.985)v^2)$$

$$= 10000(.01v + (.99)(.015)v^2 + (.99)(.985)(.02)v^3)$$

$$+ .12P + .08P(\ddot{a}_{x:\overline{3}|}) + 100$$

$$+ 30\,\ddot{a}_{x:\overline{3}|}$$

$$P = \frac{10{,}000(0.01v + (.99)(.015)v^2 + (.99)(.985)(.02)v^3) + 100 + 30(1 + .99v + (.99)(.985)v^2)}{(.92)(1 + .99v + (.99)(.985)v^2) - 0.12}$$

$$= 231.01$$

Use recursive formula for Asset Share

${}_0AS = 0$

$${}_1AS = \frac{(0 + (231.01)(.80) - 130)(1.08) - 10000(.01)}{.99}$$

$$= -41.22$$

$${}_2AS = \frac{(-41.22 - (231.01)(.92) - 30)(1.08) - 10000(.015)}{.985}$$

$$= 2.65$$

${}_3AS = 0$ since premium was

determined by equivalence principle.

We will find the benefit premiums and then use the recursive formula to find the net benefit reserves

$$Net = \frac{10000(.01v + (.99)(.015)v^2 + (.99)(.985)(.02)v^3)}{1 + .99v + (.99)(.985)v^2}$$

$= 136.13$

${}_0V^n = 0$

$${}_1V^n = \frac{(0 + 136.13)(1.08) - 10000(.01)}{.99}$$

$= 47.50$

$${}_2V^n = \frac{(47.50 + 136.13)(1.08) - (10000)(.015)}{.985}$$

$= 49.05$

${}_3V^n = 0$ by definition

Since the benefit reserve and asset shares used premiums from the equivalence principle

${}_tV^n + {}_tV^e = {}_tAS$

${}_0V^e = 0$ ${}_1V^e = -41.22 - 47.50 = -88.72$

${}_2V^e = 2.65 - 49.05 = -46.40$

${}_3V^e = 0$

103. For a fully continuous whole life of 100,000 on (50), you are given:

a. The gross premium reserve at $t = 10$ is 15,000.
b. The gross premium is paid at a rate of 2200 per year.
c. The force of interest is 8%.
d. $\mu_{50} = 0.01$
e. The following expenses payable continuously:
    i. 50% of premium in the first year and 5% of premium in years 2 and later;
    ii. 40 per policy in the first year and 20 per policy in years 2 and later; and
    iii. 500 payable at the moment of death.

Calculate the derivative of the gross premium reserve with respect to time at $t = 10$.

Calculate the annual rate of increase of the gross premium reserve at time $t = 10$..

$$\frac{d}{dt}\,{}_tV = \delta_t \cdot {}_tV + P_t - e_t - (S_t + E_t - {}_tV)\mu_{x+t} =$$

$$\frac{d}{dt}\,{}_{10}V = 0.08 \cdot 15{,}000 + 2200 - (2200 \cdot 0.05 + 20) - (100{,}000 + 500 - 15{,}000)(0.01) =$$

2415

104. For a fully continuous whole life of 100,000 on (50), you are given:

a. The gross premium reserve at $t=10$ is 15,000.
b. The gross premium is paid at a rate of 2200 per year.
c. The force of interest is 4% .
d. The force of mortality follows Gompertz law with $B=0.0015$ and $c=1.03$
e. The following expenses payable continuously:
   i. 50% of premium in the first year and 5% of premium in years 2 and later;
   ii. 40 per policy in the first year and 20 per policy in years 2 and later; and
   iii. 500 payable at the moment of death.

Estimate the gross premium reserve at $t=11$ using Euler's method with $h=0.5$.

$$ {}_{t+h}V = {}_tV + h[\delta_t \cdot {}_tV + P_t - e_t - (S_t + E_t - {}_tV)\mu_{x+t}] = $$

$$ {}_{10.5}V = {}_{10}V + 0.5[\delta_{10} \cdot {}_{10}V + P_{10} - e_{10} - (S_{10} + E_{10} - {}_{10}V)\mu_{50+10}] = $$

$$ 15,000 + 0.5[0.04 \cdot 15,000 + 2200 - (2200 \cdot 0.05 + 20) - (100,000 + 500 - 15,000)\{(0.0015)(1.03)^{60}\}] = $$

$$ 15,957.20 $$

$$ {}_{11}V = {}_{10.5}V + 0.5[\delta_{10.5} \cdot {}_{10.5}V + P_{10.5} - e_{10.5} - (S_{10.5} + E_{10.5} - {}_{10.5}V)\mu_{50+10.5}] = $$

$$ 15,957.20 + 0.5[0.04 \cdot 15,957.20 + 2200 - (2200 \cdot 0.05 + 20) - (100,000 + 500 - 15,957.20)\{(0.0015)(1.03)^{60.5}\}] = $$

$$ 16,932.08 $$

105. For a fully continuous 10 year term insurance issued to age 70, you are given:

a. The death benefit if 250,000.
b. The net benefit premium is paid at a rate of 13,000 per year.
c. The force of interest is 6% .
d. The force of mortality follows Makeham's law with $A = 0.005$, $B = 0.002$ and $c = 1.05$

Estimate the net premium reserve at $t = 9.5$ using Euler's method with $h = 0.25$.

$$ {}_tV = \frac{{}_{t+h}V - h[P_t - e_t - (S_t + E_t)\mu_{x+t}]}{1 + h[\delta_t + \mu_{x+t}]} $$

$$ {}_{9.75}V = \frac{{}_{10}V - 0.25[P_{9.75} - e_{9.75} - (S_{9.75} + E_{9.75})\mu_{70+9.75}]}{1 + 0.25[\delta_{9.75} + \mu_{70+9.75}]} = $$

$$ \frac{0 - 0.25[13{,}000 - 0 - (250{,}000 + 0)\{0.005 + (0.002)(1.05)^{79.75}\}]}{1 + 0.25[0.06 + \{0.005 + (0.002)(1.05)^{79.75}\}]} = $$

3058.02

$$ {}_{9.5}V = \frac{{}_{9.75}V - 0.25[P_{9.5} - e_{9.5} - (S_{9.5} + E_{9.5})\mu_{70+9.5}]}{1 + 0.25[\delta_{9.5} + \mu_{70+9.5}]} = $$

$$ \frac{3058.02 - 0.25[13{,}000 - 0 - (250{,}000 + 0)\{0.005 + (0.002)(1.05)^{79.5}\}]}{1 + 0.25[0.06 + \{0.005 + (0.002)(1.05)^{79.5}\}]} = $$

5926.76

106. For a fully continuous 10 endowment insurance issued to age 70, you are given:

a. The death benefit if 250,000.
b. The net benefit premium is paid at a rate of P per year.
c. The force of interest is 6%.
d. The force of mortality follows Makeham's law with $A = 0.005$, $B = 0.002$ and $c = 1.05$
e. The net premium reserve at $t = 9.5$ using Euler's method with $h = 0.25$ is estimated to be 230,000.

Calculate P.

$$ {}_tV = \frac{{}_{t+h}V - h[P_t - e_t - (S_t + E_t)\mu_{x+t}]}{1 + h[\delta_t + \mu_{x+t}]} $$

$$ {}_{9.75}V = \frac{{}_{10}V - 0.25[P_{9.75} - e_{9.75} - (S_{9.75} + E_{9.75})\mu_{70+9.75}]}{1 + 0.25[\delta_{9.75} + \mu_{70+9.75}]} = $$

$$ \frac{250,000 - 0.25[P - 0 - (250,000 + 0)\{0.005 + (0.002)(1.05)^{79.75}\}]}{1 + 0.25[0.06 + \{0.005 + (0.002)(1.05)^{79.75}\}]} = $$

$$ \frac{256,432.5732 - 0.25P}{1.040730293} = 246,396.7609 - 0.240215935P $$

$$ {}_{9.5}V = \frac{{}_{9.75}V - 0.25[P_{9.5} - e_{9.5} - (S_{9.5} + E_{9.5})\mu_{70+9.5}]}{1 + 0.25[\delta_{9.5} + \mu_{70+9.5}]} = $$

$$ 230,000 = \frac{246,396.7609 - 0.240215935P - 0.25[P - 0 - (250,000 + 0)\{0.005 + (0.002)(1.05)^{79.5}\}]}{1 + 0.25[0.06 + \{0.005 + (0.002)(1.05)^{79.5}\}]} $$

$$ P = \frac{230,000(1 + 0.25[0.06 + \{0.005 + (0.002)(1.05)^{79.5}\}]) - 246396.7609 - 0.25(250,000)\{0.005 + (0.002)(1.05)^{79.5}\}}{-0.240215935 - 0.25} = $$

$$ \frac{-13455.43103}{-0.490215935} = 27,447.97 $$